Konzultace k 10. přednášce

$$y[n+2] = y[n+1] + y[n] \qquad y[0]=1 \;;\; y[1]=1$$

$$\mathcal{Z}\{y[n-m]\} = z^{-m}\,\mathcal{Z}\{y[n]\} = z^{-m}\cdot Y(z)$$
$$\mathcal{Z}\{y[n+m]\} = z^{m}\left[Y(z) - \sum_{v=0}^{m-1} y[v]\cdot z^{-v}\right]$$

$$z^2\left[Y(z) - y[0]\cdot z^{-0} - y[1]\cdot z^{-1}\right] = z\left[Y(z) - y[0]\cdot z^{-0}\right] + Y(z)$$
$$z^2\left[Y(z) - 1 - \frac{1}{z}\right] = z\left[Y(z) - 1\right] + Y(z)$$
$$z^2 Y(z) - z^2 - \cancel{z} = z\,Y(z) - \cancel{z} + Y(z)$$

$$Y(z)\left[z^2 - z - 1\right] = z^2$$

$$Y(z) = \frac{z^2}{z^2 - z - 1}$$

$$z^2 - z - 1$$

$$z_{1,2} = \frac{1 \pm \sqrt{1+4}}{2} = \frac{1\pm\sqrt{5}}{2}$$

$$z_1 = \frac{1+\sqrt{5}}{2} \;;\; z_2 = \frac{1-\sqrt{5}}{2}$$

$$Y(z) = \frac{z^2}{(z-z_1)(z-z_2)}$$

$$\frac{Y(z)}{z} = \frac{z}{(z-z_1)(z-z_2)}$$

$$\frac{z}{(z-z_1)(z-z_2)} = \frac{A}{z-z_1} + \frac{B}{z-z_2}$$

$$A = \frac{z}{z-z_2}\Big|_{z=z_1} = \frac{z_1}{z_1-z_2}$$

$$B = \frac{z}{z-z_1}\Big|_{z=z_2} = \frac{z_2}{z_2-z_1}$$

$\dfrac{z}{z-1}$	
$\dfrac{z}{z-a}$	
$\dfrac{z}{(z-a)^2}$	
$\dfrac{z^2}{(z-a)^2}$	
$\dfrac{z}{(z-1)^2}$	
$\dfrac{z(z+1)}{(z-1)^3}$	

$$Y[z] = \frac{z_1}{z_1-z_2}\cdot\frac{z}{z-z_1} + \frac{z_2}{z_2-z_1}\cdot\frac{z}{z-z_2}$$
$$Y[z] = \frac{1}{z_1-z_2}\left[z_1\cdot\frac{z}{z-z_1} - z_2\cdot\frac{z}{z-z_2}\right]$$
$$y[n] = \frac{1}{z_1-z_2}\left(z_1\cdot z_1^{n} - z_2\cdot z_2^{n}\right)$$

a^n	$\dfrac{1}{1-az^{-1}}$	$\dfrac{z}{z-a}$

$$y[n] = \frac{1}{z_1-z_2}\left(z_1^{n+1} - z_2^{n+1}\right)$$

I. Přenosová funkce

$$a_N\cdot y[n-N] + a_{N-1}\,y[n-N+1] + \dots + a_0\cdot y[n] = b_M\cdot u[n-M] + b_{M-1}\cdot u[n-M+1] + \dots + b_0\cdot u[n]$$

$$\sum_{k=0}^{N} a_k\, y[n-k] = \sum_{\ell=0}^{M} b_\ell\, u[n-\ell]$$

$$\mathcal{Z}\left\{\sum_{k=0}^{N} a_k\cdot y[n-k]\right\} = \mathcal{Z}\left\{\sum_{\ell=0}^{M} b_\ell\cdot u[n-\ell]\right\}$$

$$\sum_{k=0}^{N} a_k\cdot \mathcal{Z}\{y[n-k]\} = \sum_{\ell=0}^{M} b_\ell\cdot\mathcal{Z}\{y[n-\ell]\}$$

$$\sum_{k=0}^{N} a_k\cdot z^{-k}\cdot Y(z) = \sum_{\ell=0}^{M} b_\ell\cdot z^{-\ell}\cdot U(z)$$

Přenosová funkce

$$H(z) = \frac{Y(z)}{U(z)}$$; nulové pp.

$$H(z) = \frac{Y(z)}{U(z)} = \frac{\sum_{\ell=0}^{M} b_\ell\cdot z^{-\ell}}{\sum_{k=0}^{N} a_k\cdot z^{-k}} = \frac{Q(z)}{N(z)}$$

$$y[n] = \sum_{m=0}^{n} h(n-m)\cdot u(n)$$

II. Impulsní odezva

$$h[n] = \mathcal{Z}^{-1}\{H(z)\}$$

II. Impulsní odezva

$$h[n] = \mathcal{Z}^{-1}\{H(z)\}$$

III. Přechodová odezva

$$S(z) = H(z) \cdot \mathcal{Z}\{\mathbb{1}[n]\} = H(z) \cdot \frac{1}{1-z^{-1}} = H(z) \cdot \frac{z}{z-1}$$

$$s[n] = \mathcal{Z}^{-1}\{S(z)\}$$

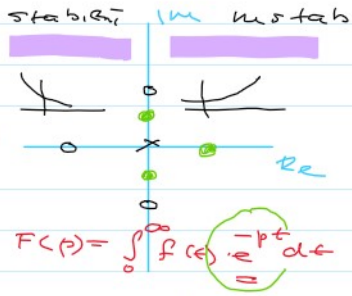
IV. Stabilita diskrétních systémů

$$y[n+2] + a \cdot y[n+1] + b \cdot y[n] = u[n] \quad ; \quad \text{pro nulové p.p.}$$

$$H(z) = \frac{1}{z^2 + az + b} = \frac{1}{(z-z_1)(z-z_2)}$$

spojité systémy

stabilní Im nestab.



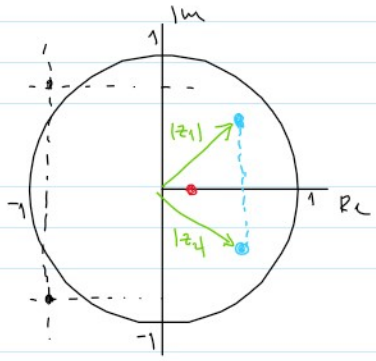
$$F(p) = \int_0^{\infty} f(t) \cdot e^{-pt} \, dt$$

Př. z-transf.

$$F(z) = \sum_{n=0}^{\infty} f[n] \cdot z^{-n} \qquad \Big\} \lim_{n \to \infty} h[n]$$

$$\lim_{n \to \infty} q^n \begin{cases} 0 & ; q < 1 \quad q = \frac{1}{2} \; \{ \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \dots \\ 1 & ; q = 1 \quad \{1, 1, 1, \dots\} \\ \infty & ; q > 1 \quad q = 2 \; \{2, 4, 8, 16 \dots\} \end{cases}$$

$$H(z) = \frac{1}{(z-z_1)(z-z_2)} \quad ; \quad h[n] = k_1 \cdot (z_1)^n + k_2 \cdot (z_2)^n$$

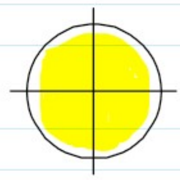


$$z_{1,2} = Re \mp Im$$

$$|z_{1,2}| < 1$$

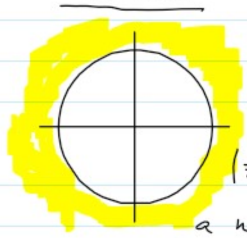
$$|z_{1,2}| = \sqrt{Re^2 + Im^2}$$

Stabilní



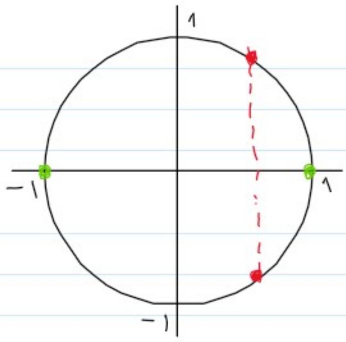
$|z_{1,2}| < 1$

Nestabilní



$|z_{1,2}| > 1$ a nebo násobné kořeny $|z_{1,2}| = 1$

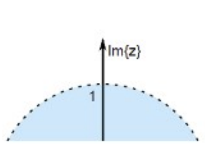
Mez stability

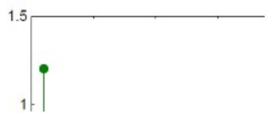


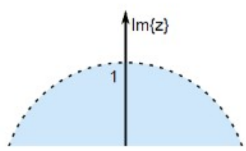
a) nenásobné reálné kořeny

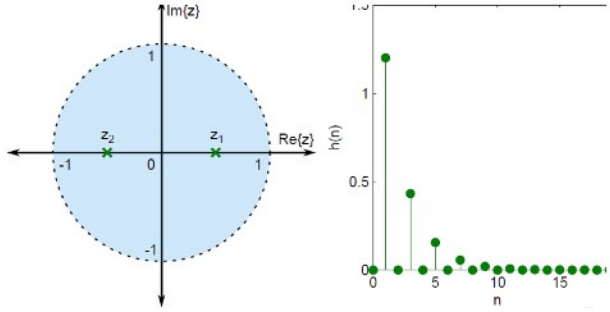
b) nenásobné komplexní kořeny $|z_{1,2}| = 1$

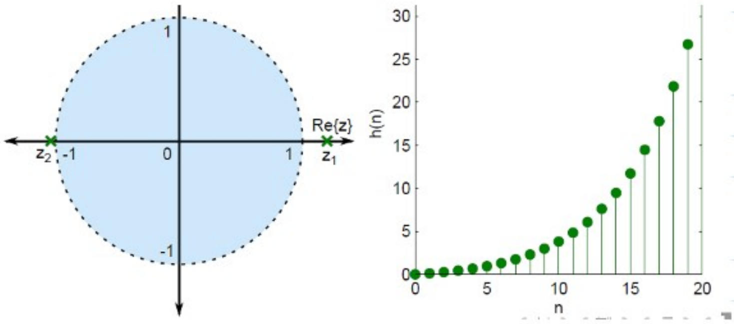
Příklad:

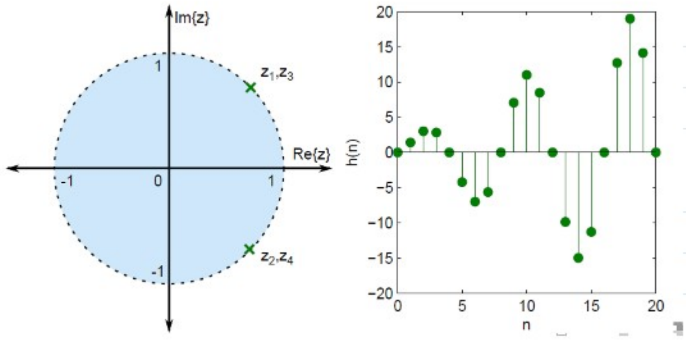


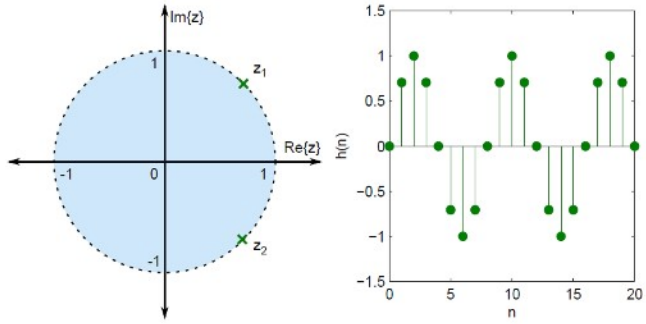


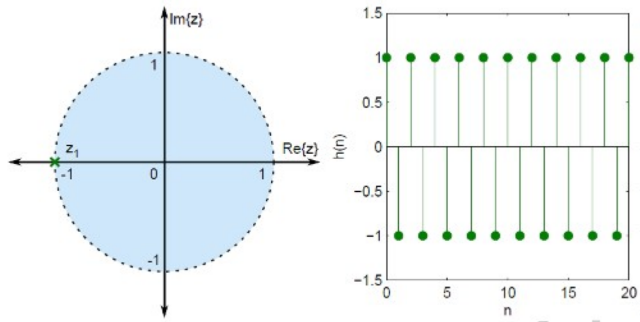


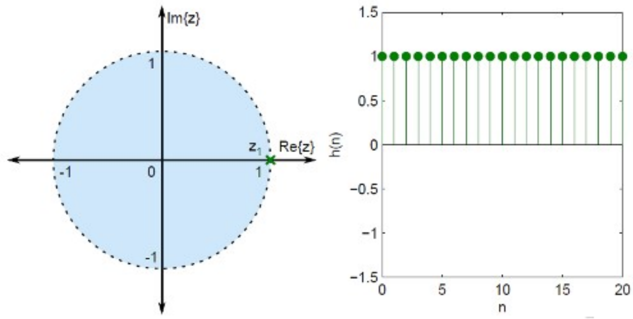












Stabilita pro vnitřní popis

$$x[n+1] = M\,x[n] + N \cdot u[n]$$
$$y[n] = C \cdot x[n]$$

Z-transformace

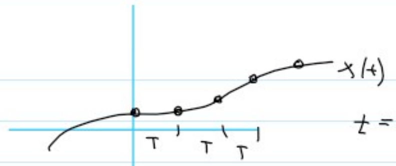
$$z\left(X(z) - x[0]\right) = M\,X(z) + N\,U(z)$$
$$Y(z) = C \cdot X(z)$$

$$z\,X(z) = M\,X(z) + N\,U(z)$$
$$X(z) = (z\underline{I} - M)^{-1} N\,U(z)$$

$$Y(z) = C\,(z \cdot \underline{I} - M)^{-1} N\;U(z)$$

$$H(z) = \mathbf{C}\,(z\mathbf{I} - \mathbf{M})^{-1}\mathbf{N} = \mathbf{C}\,\frac{\operatorname{adj}(z\mathbf{I} - \mathbf{M})}{\det(z\mathbf{I} - \mathbf{M})}\,\mathbf{N}$$

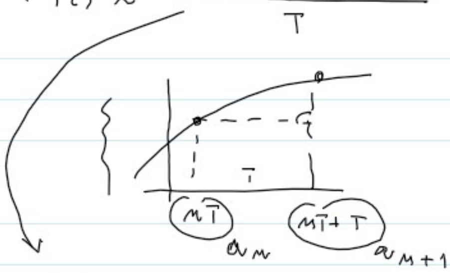
$\det(z\underline{I} - M)$



$t = nT$

$$x(t) = x(nT) = x[n]$$

$$x'(t) \approx \frac{x(nT + T) - x(nT)}{T}$$



$$x'(t) = A \cdot x(t) + B\,u(t)$$
$$y(t) = C \cdot x(t) + D \cdot u(t)$$

$$x'(t) \approx \frac{x(nT+T) - x(nT)}{T} = \frac{1}{T}\left(x[n+1] - x[n]\right)$$

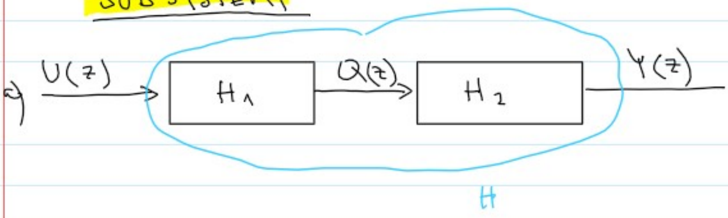
$$\frac{1}{T}\left(x[n+1] - x[n]\right) = A\,x[n] + B\,u[n]$$

$$\frac{1}{T}\left(x[n+1]-x[n]\right) = Ax[n] + Bu[n]$$
$$x[n+1]-x[n] = TAx[n] + TBu[n]$$
$$x[n+1] = Ix_n + TAx[n] + TBu[n]$$
$$x[n+1] = (1+TA)x[n] + TBu[n]$$
$$y[n] = Cx[n] + Du[n$$

$$x[n+1] = M\cdot x[n] + Nu[n]$$
$$y[n] = Cx[n] + Du[n]$$

SUBSYSTÉMY

a)



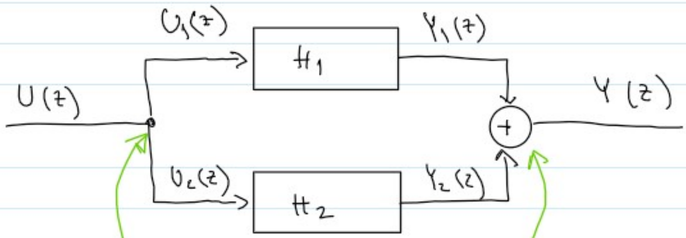
$$H(z) = \frac{Y(z)}{U(z)}$$
$$H_1(z) = \frac{Q(z)}{U(z)} \Rightarrow U(z) = \frac{Q(z)}{H_1(z)}$$
$$H_2(z) = \frac{Y(z)}{Q(z)} \Rightarrow Y(z) = H_2(z)\cdot Q(z)$$

$$H(z) = \frac{H_2(z)\cdot Q(z)}{\frac{Q(z)}{H_1(z)}}$$

$$\boxed{H(z) = H_1(z)\cdot H_2(z)}$$

kaskáda (seriové zapojení)

b)



$$H(z) = \frac{Y(z)}{U(z)}$$
$$H_1(z) = \frac{Y_1(z)}{U_1(z)}$$
$$H_2(z) = \frac{Y_2(z)}{U_2(z)}$$
$$U_1(z) = U_2(z) = U(z)$$

$$U(z) = U_1(z) = U_2(z) \qquad Y(z) = Y_1(z) + Y_2(z)$$

$$H(z) = \frac{Y(z)}{U(z)} = \frac{Y_1(z)+Y_2(z)}{U(z)} = \frac{Y_1(z)}{U(z)} + \frac{Y_2(z)}{U(z)} = H_1(z) + H_2(z)$$

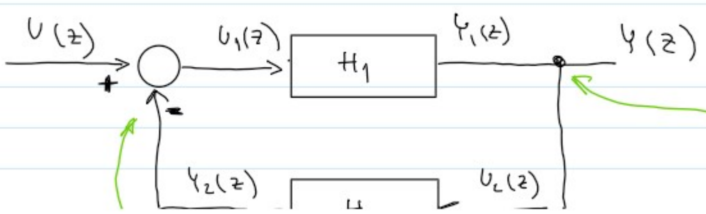
$$\boxed{H(z) = H_1(z) + H_2(z)}$$

Palalelní zapojení

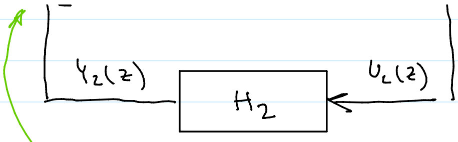
c) zpětná vazba

(i) záporná

$$H(z) = \frac{Y(z)}{U(z)}$$



výstup $Y_1(z) = Y(z) = U_2(z)$



výstup $Y_1(z) = Y(z) = U_2(z)$

vstup $U_1(z) = U(z) - Y_2(z)$

$$H_1(z) = \frac{Y_1(z)}{U_1(z)} = \frac{Y(z)}{U_1(z)} \Rightarrow Y(z) = H_1(z) \cdot U_1(z)$$

$$Y(z) = H_1(z)\left[U(z) - Y_2(z)\right] =$$

$$H_2(z) = \frac{Y_2(z)}{U_2(z)}$$

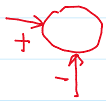
$$\Rightarrow Y_2(z) = H_2(z) \cdot U_2(z)$$

$$= H_1(z)\left[U(z) - H_2(z) \cdot U_2(z)\right] =$$
$$= H_1(z)\left[U(z) - H_2(z) \cdot Y(z)\right]$$

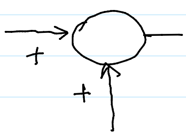
$$Y(z) = H_1(z)\left[U(z) - H_2(z) \cdot Y(z)\right]$$
$$Y(z) = H_1(z) \cdot U(z) - H_1(z) \cdot H_2(z) \cdot Y(z)$$
$$Y(z)\left[1 + H_1(z) H_2(z)\right] = H_1(z) \cdot U(z)$$

$$\boxed{H(z) = \frac{Y(z)}{U(z)} = \frac{H_1(z)}{1 + H_1(z) \cdot H_2(z)}}$$

záporná zpětná vazba

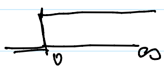


ii) kladná zpětná vazba

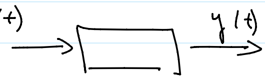


$$\boxed{H(z) = \frac{H_1(z)}{1 - H_1(z) \cdot H_2(z)}}$$

Konzultace



$y(t) = 2e^{-3t} \cdot \mathbb{1}(t)$ $\qquad u(t) = \mathbb{1}(t)$



$Y(p) = \mathcal{L}\left\{2 \cdot e^{-3t} \cdot \mathbb{1}(t)\right\}$ $\qquad U(p) = \frac{1}{p}$ $\qquad H(p) = \frac{Y(p)}{U(p)}$

$2 \cdot e^{-3t} \cdot 1$

$$Y(p) = \frac{2}{p+3} \qquad H(p) = \frac{\frac{2}{p+3}}{\frac{1}{p}} = \frac{2p}{p+3} = 2 - \frac{6}{p+3}$$

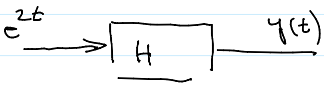
$$2\left(\frac{p+3}{p+3} + \frac{-3}{p+3}\right)$$

a) $h(t) = 2 \cdot \delta(t) - 6 \cdot e^{-3t}$

b) $$S(p) = \frac{H(p)}{p} = \frac{\frac{2p}{p+3}}{p} = \frac{2}{p+3}$$

$$s(t) = 2 \cdot e^{-3t}$$

c)



$$H(p) = \frac{Y(p)}{U(p)} \rightarrow Y(p) = H(p) \cdot U(p)$$
$$= \frac{2p}{p+3} \cdot \frac{1}{p-2}$$

$$H(p) = \frac{Y(p)}{U(p)} \Rightarrow Y(p) = H(p) \cdot U(p)$$

$$= \frac{2p}{p+3} \cdot \frac{1}{p-2}$$